Carderock Division

Naval Surface Warfare Center

West Bethesda, Maryland 20817-5700

NSWCCD-50-TR-2000 / 040 September 2000

Hydromechanics Directorate

Research and Development Report

Dynamometer Calibration and Usage

by

David E. Hess
Richard T. Nigon
John W. Bedel

Approved for public release: distribution unlimited.

REPORT DOCUMENTATION PAGE

Form Approved OMB No. 0704-0188

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	September 2000	Final

4. TITLE AND SUBTITLE
Dynamometer Calibration and Usage

5. FUNDING NUMBERS
N0002400WR10453
PE 64558N
W/U 1-5600-259-50

6. AUTHOR(S)
David E. Hess, Richard T. Nigon and John W. Bedel

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Carderock Division, Naval Surface Warfare Center
Code 5600
West Bethesda, Maryland 20817-5700

8. PERFORMING ORGANIZATION REPORT NUMBER
NSWCCD-50-TR-2000 / 040

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Mr. Matthew B. King (PMS 450) 703-602-1400 x110
Naval Sea Systems Command - 05H2
2531 Jefferson Davis Highway
Arlington, VA 22242-5160

10. SPONSORING / MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION / AVAILABILITY STATEMENT
Approved for public release: distribution unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

This report describes a potpourri of information concerning dynamometers that has been in use by various groups within the Maneuvering and Control Department of the Hydrodynamics Directorate. Because these transducers are complex devices requiring care with their calibration and usage, the authors felt that the important details should be recorded and made available to others at the Center. Following a basic discussion of the operation of the transducer and the need for an interaction matrix is a suggested calibration procedure that will yield data that completely describes the six degree-of-freedom parameter space of the unit. Subsequent sections describe numerical methods for the derivation of the interaction matrix from calibration data. The first of these methods is a Least Squares solution for the interaction matrix, and the second is a generalization of the first method that allows weighting of the data. Other topics include: a method for transformation of an interaction matrix determined relative to one coordinate system into an equivalent interaction matrix relative to a second coordinate system, and a technique for normalizing the interaction matrix. Additional complications arise when dealing with rotating dynamometers and such topics as determination of the offset angle, and weight and zero compensation are investigated. One may wish to use two dynamometers in tandem to determine forces and moments about a common reference point; the equations required for such operation are described in detail. A discussion of data uncertainty is incorporated, and finally, an appendix is included which describes how interaction matrices obtained from a dynamometer manufacturer were found to be in error, and how the methods described in this paper were used to detect and correct the errors.

14. SUBJECT TERMS
Dynamometer calibration, dynamometer usage, interaction matrix

15. NUMBER OF PAGES
37

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	Unclassified

THIS PAGE INTENTIONALLY LEFT BLANK

Contents

Abstract 1
Administrative Information 1
Overview 1
Static Calibration Procedure 3
Load Placement 5
Pseudo-Inverse Technique 7
Weighted Least Squares Derivation (Pseudo-Inverse) 8
Coordinate System Transformation 10
Normalization Technique 13
Rotating Dynamometer Offset Angle 14
Rotating Dynamometer Weight and Zero Compensation 16
Equations for Utilizing Two Six Degree-of-Freedom Dynamometers 19
Data Uncertainty 20
Conclusions 24
Appendix 25
References 29

Figures

1. Dynamometer coordinate system (from AMTI). 4
2. Thrust axis load placement. 5
3. Side force load placement. 6
4. Dynamometer coordinate system (DCS) on left and standard coordinate system on right. 10
5. Two dynamometers aligned along their F_x axis and equidistant from the reference center. 19

Tables

1. Normalized interaction matrix. 14
2. Inverse of the gain matrix. 14
3. Original interaction matrix. 14
4. Uncorrected and corrected interaction matrices. 25

Abstract

This report describes a potpourri of information concerning dynamometers that has been in use by various groups within the Maneuvering and Control Department of the Hydrodynamics Directorate. Because these transducers are complex devices requiring care with their calibration and usage, the authors felt that the important details should be recorded and made available to others at the Center. Following a basic discussion of the operation of the transducer and the need for an interaction matrix is a suggested calibration procedure that will yield data that completely describes the six degree-of-freedom parameter space of the unit. Subsequent sections describe numerical methods for the derivation of the interaction matrix from calibration data. The first of these methods is a Least Squares solution for the interaction matrix, and the second is a generalization of the first method that allows weighting of the data. Other topics include: a method for transformation of an interaction matrix determined relative to one coordinate system into an equivalent interaction matrix relative to a second coordinate system, and a technique for normalizing the interaction matrix. Additional complications arise when dealing with rotating dynamometers and such topics as determination of the offset angle, and weight and zero compensation are investigated. One may wish to use two dynamometers in tandem to determine forces and moments about a common reference point; the equations required for such operation are described in detail. A discussion of data uncertainty is incorporated, and finally, an appendix is included which describes how interaction matrices obtained from a dynamometer manufacturer were found to be in error, and how the methods described in this paper were used to detect and correct the errors.

Administrative Information

This work was sponsored by PMS 450 under Contract N0002400WR10453 and Program Element 64558N. The program representative is Mr. Matthew B. King.

Overview

A six degree-of-freedom dynamometer is a transducer capable of simultaneously measuring three force components, denoted by F_x, F_y, and F_z and referred to as thrust, side force and normal force, and three moments, denoted by M_x, M_y, and M_z and referred to as torque, Y bending moment and Z bending

moment when using the standard NSWC coordinate system[1,2]. For the work discussed here, the dynamometer is assumed to be mounted on a portion of a vehicle moving through a fluid and measures the forces and moments exerted by the fluid on this portion of the vehicle. These six quantities are measured relative to a coordinate system fixed within the dynamometer and defined by the placement of the various strain gages within the unit which transduce the force and moment measurements into voltages. The six output voltages of the dynamometer can be converted back into dimensional quantities (*lbs* and *ft-lbs*) by using an appropriate method that requires calibration data.

When a perfect dynamometer is subjected to a known load on one axis only, a known thrust force for example, it will produce a single nonzero output voltage and the other five voltage outputs will remain zero. In other words, the outputs of a perfect dynamometer represent six independent measurements. On the other hand, for most real dynamometers, a known load on a single axis produces six nonzero voltage outputs; typically, one primary responding output and five smaller outputs resulting from *interactions* (possibly nonlinear) among the transducing elements within the dynamometer. If these six outputs were then interpreted as originating from a perfect dynamometer, then one would *incorrectly* predict that the dynamometer was subjected to loads on all six axes. Instead, one must correct for the interaction effects, and the correction may be implemented using a linear or nonlinear method.

For a six degree-of-freedom dynamometer, the linear correction method takes the form of a 6 x 6 matrix routinely termed the *interaction matrix*. The determination of this matrix for a real dynamometer is an additional task, which must be completed during calibration. A suitable numerical method for the derivation of the interaction matrix from the calibration data is described in a subsequent section as is a similar but more complex method using weighted calibration data. If we group the measurements in the order: F_x, F_y, F_z, M_x, M_y and M_z, and then denote this ordering with subscripts from 1 to 6, then the interaction matrix multiplies the six output voltages from the dynamometer, V_i, to obtain six corrected dimensional quantities, F_i or (F_x, F_y, F_z, M_x, M_y and M_z), as shown in Eq. 1:

$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} & A_{16} \\ A_{21} & A_{22} & A_{23} & A_{24} & A_{25} & A_{26} \\ A_{31} & A_{32} & A_{33} & A_{34} & A_{35} & A_{36} \\ A_{41} & A_{42} & A_{43} & A_{44} & A_{45} & A_{46} \\ A_{51} & A_{52} & A_{53} & A_{54} & A_{55} & A_{56} \\ A_{61} & A_{62} & A_{63} & A_{64} & A_{65} & A_{66} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} \quad \text{or} \quad F_i = A_{ij} V_j \text{ for } i, j = 1, \ldots, 6. \qquad (1)$$

The repeated index implies summation in Eq 1.

Summarizing, then, the calibration of a dynamometer is performed in order to determine a set of output voltages for a series of known single axis or multiple axis loading conditions. Using this data, one may compute an interaction matrix, A_{ij}, which gives a linear approximation of the behavior of the dynamometer.

Alternatively, recent work has shown that a nonlinear method, such as an artificial neural network, can also accurately recover the calibration data. The possibility that a neural network can provide a nonlinear correction to the linear approximation afforded by the interaction matrix should be investigated. However, because a dynamometer is primarily linear in its response to loads the utility of the neural network for this application will have to be clearly demonstrated. Such work is in progress.

Static Calibration Procedure

The static calibration of six degree of freedom dynamometers that exhibit interactions must be performed in a manner that fully captures the intricacies of the complex behavior of these devices. Experience has shown that a dynamometer must be subjected to a substantial number of instances of single axis and multiple axis loading conditions in order to completely characterize its performance. Furthermore, the selection of the locations of the applied loads is extremely important to define the six degree of freedom parameter space that governs the behavior of the dynamometer. The following paragraphs will outline a static calibration procedure expected to give good results. Note that dynamometers exposed to unsteady loading conditions varying over an extended frequency range should be calibrated *dynamically* to ensure adequate frequency response and to avoid resonances in the measuring system[3].

Each channel of the dynamometer will respond with a voltage proportional to the applied load. However, these output voltages are also linearly proportional to the excitation voltage V_{ex} used to power the dynamometer. Therefore, output voltages are typically expressed in terms of $\mu V/V_{ex}$ in order to yield results that are independent of the excitation voltage. During calibration, the output voltages for each loading condition will be digitized by an A/D converter to give bits that must then be converted back into voltages using the appropriate factor for the A/D converter. For purposes of illustration, assume a 12-bit A/D converter, which digitizes a $\pm 10\mathrm{V}$ range with 4096 bits. Assume also that the voltages from each channel are amplified by an amount specified by a gain factor. For these conditions Eq. 2 specifies the conversion from bits to $\mu V/V_{ex}$ for each channel.

$$\frac{\mu V}{V_{ex}} = bits * \left(\frac{20}{4096}\frac{V}{bit}\right) * \left(1000000\frac{\mu V}{V}\right) * \left(\frac{1}{Gain}\right) * \left(\frac{1}{V_{ex}}\right) , \qquad (2)$$

Therefore, for each loading condition, the A/D conversion factor, the amplifier gain (if used) for each channel and the excitation voltage *must* be specified in order to recover the output voltages in $\mu V/V_{ex}$. If an alternative system is used, then all necessary information required to recover the output voltages in $\mu V/V_{ex}$ must be given for each loading condition.

The application of a load at a given location on a six degree of freedom dynamometer is stated in terms of three force components and three moment components referred to a coordinate system fixed within the dynamometer. The coordinate system that will be used to define the loading conditions is one commonly used by dynamometer manufacturers and is shown in Fig. 1. An interaction matrix determined relative to this or any other coordinate system may be easily transformed into one relative to

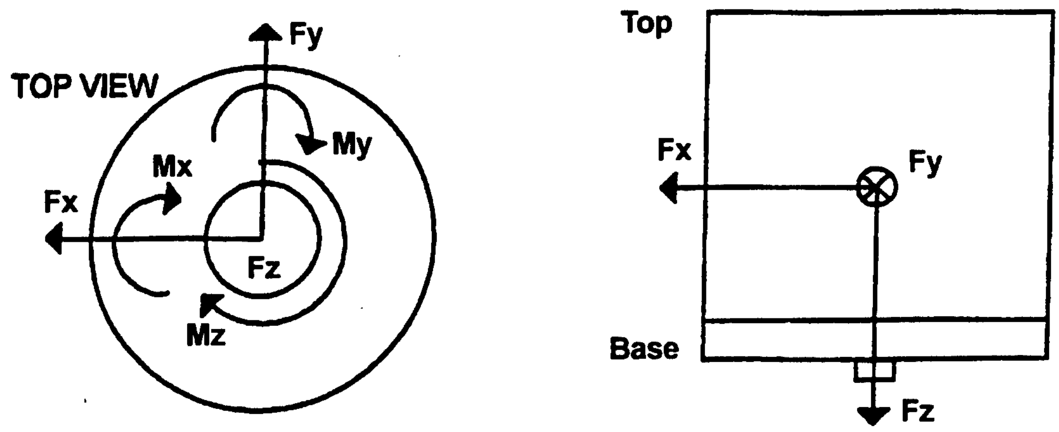

Fig. 1 Dynamometer coordinate system (from AMTI).
Z axis into the paper in left hand sketch.

the more familiar 2510 coordinate system[1,2] by using a method described in a later section. Therefore, the coordinate system described in Fig. 1 will be used for this section only, and the standard NSWC coordinate system[1,2] will be used for the remainder of the paper.

The origin of the coordinate system is determined as the point where the application of forces produces zero moments. This location must be determined during calibration and specified relative to some unambiguous and readily identifiable point on the unit such as the geometric center of the top face. These offset distances will be denoted by x_0, y_0 and z_0, respectively. On the other hand, the application of a load on a dynamometer may be specified relative to an alternative set of axes more useful for the problem at hand. The applied forces will not depend on the location of the origin but the applied moments will have to be transformed to the new location of the origin. If x_1, y_1 and z_1, represent offsets from the internal origin to the new user-defined origin, then Eqs. 3 define the moments about the new origin in terms of the moments about the internal origin.

$$\begin{aligned} M_x &= M_{x0} - F_y z_1 + F_z y_1 \\ M_y &= M_{y0} - F_z x_1 + F_x z_1 \ , \\ M_z &= M_{z0} - F_x y_1 + F_y x_1 \end{aligned} \tag{3}$$

During calibration, the applied loads for each loading condition will have to be specified. Often they are most readily specified relative to some easily identifiable location on the unit such as the geometric center of the top face. However, they should properly be referred to the internal origin of the unit. For this case, one must invert Eqs. 3 and solve for M_{x0}, M_{y0} and M_{z0}.

Load Placement

The calibration proceeds by placing a force on the unit at a given location and measuring the output voltages. The magnitude of the force (at the same location) is then changed to obtain the next load condition. This sequence is repeated for a series of 5-10 force magnitudes linearly increasing up to some predetermined calibration maximum. Then, the location is changed and the previous process is repeated. For a rotor or stator dynamometer as typically used in the Maneuvering and Control Department, the maximum expected thrust is 150 pounds, and the maximum expected side loads are 50 pounds. Accordingly, the maximum loads applied during calibration should only be a small percentage higher than this. Now, at each location the applied load typically consists of a force component and one or two moment components. Of paramount importance, is the selection of the locations at which to apply loads such that the resulting force and moment combinations completely describe the six degree-of-freedom parameter space of the unit.

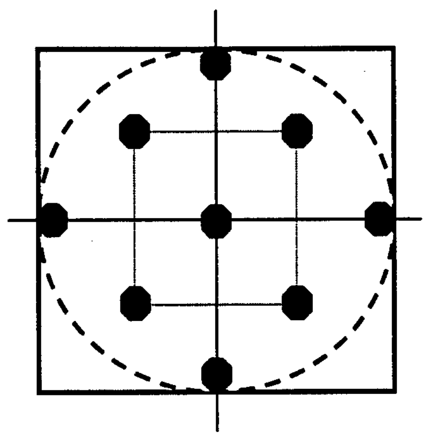

Fig. 2 Thrust axis load placement.

Beginning with loads on the thrust axis (z-axis) of the unit, a series of locations at which loads will be placed are specified in Fig. 2. The locations shown in black are required, and the locations shown in gray are desirable. A force directed into the paper at the positions denoted by black circles will result in combinations of a positive thrust and one positive or negative moment about the x and y axes. Forces applied at the gray locations will yield combinations of a force and two moments with different moment arms about the x and y axes. A means for applying forces to the unit in the negative thrust direction must also be found. The simplest manner in which this may be accomplished may perhaps be to invert the unit and apply the loads indicated in Fig. 2 to the bottom face of the unit. A connector is typically located on the bottom face, and if the connector interferes with any of the load positions, then this position will have to be skipped.

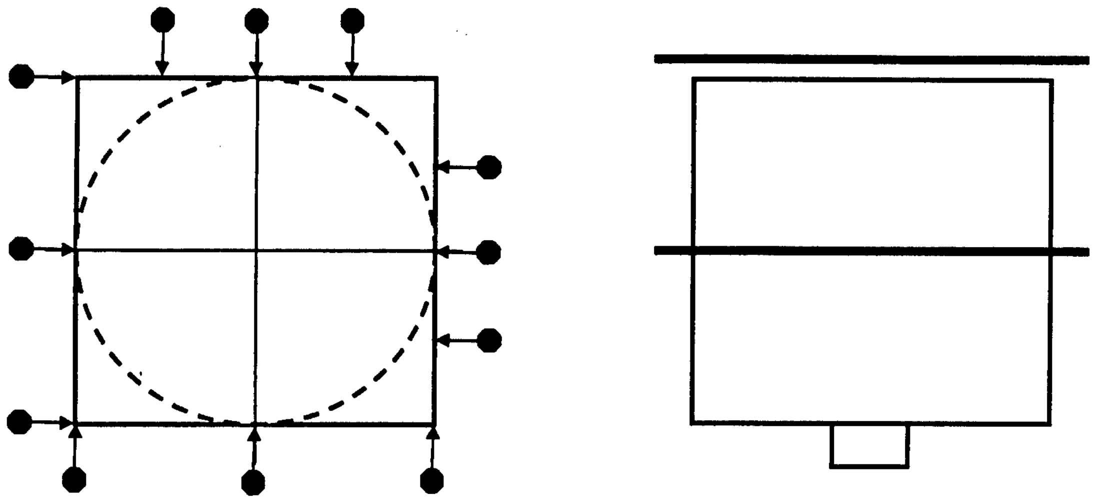

Fig. 3 Side force load placement.

Side force loads will be placed on the unit at the locations shown in black on the left of Fig. 3. These positions may be accommodated by affixing a plate of known dimensions to the top of the unit. A force directed into the paper at the positions denoted by black circles will result in combinations of a positive side force and one or two positive or negative moments about the x or y and z-axes. This set of locations lies in a plane that is located at the center of the thickness dimension of the plate attached to the top of the unit. Therefore, the plane lies some small distance above the top of the unit as shown on the right of Fig. 3. All of the moments about the x or y axes which result from the locations in this plane share a constant moment arm in the z-direction relative to the dynamometer origin. An additional set of measurements with a different length moment arm in the z-direction is desirable to fully describe the behavior of the dynamometer. This can be accomplished by repeating the set of measurement locations described on the left of Fig. 3 with the plane of the measurement locations moved to a second axial position as shown on the right of Fig. 3. The desired position of this second plane will be provided. A plate with a center hole, which accommodates the body of the dynamometer, should facilitate the acquisition of the second plane of data. Acquiring static calibration data at multiple axial locations has also been performed by Miller[3].

Summarizing, Figs. 2 and 3 describe a set of as many as 42 measurement locations with 5-10 load conditions acquired at each location by varying the magnitude of the applied force. This results in a total of 240 to 420 load conditions for the calibration. These positions should be sufficient to fully describe the complex behavior of the unit. The measured voltages, A/D conversion factor, the amplifier gain (if used) for each channel, the excitation voltage and applied loads for each of the 240 to 420 load conditions must be provided in an electronic format (text files with comma or space delimitation).

After a suitable set of calibration data is obtained, applied forces and moments along with corresponding output voltages, the information is used to construct the interaction matrix for the unit. A general linear method to accomplish this is provided in the next section.

Pseudo Inverse Technique

This appendix describes a *linear* method for the determination of an interaction matrix from calibration data. This method makes use of *all* of the calibration data and determines the best interaction matrix in a Least Squares sense.

Given a set of N instances of dynamometer measurements with M measurements for each instance, the data must satisfy:

$$\mathbf{F} = \mathbf{V}\mathbf{A} \quad , \tag{4}$$

where **F** is the set of forces and moments that produces the voltage outputs **V**. **F** is an $N \times M$ matrix of input forces and moments, **V** is an $N \times M$ matrix of resulting voltages, and **A** is an $M \times M$ matrix. Note that in this formulation the data are in row format with each row representing a separate instance of the calibration process.

The pseudo-inverse of **V** can be calculated by computing:

$$\text{Pseudo-inverse} = \left(\mathbf{V}^T \mathbf{V}\right)^{-1} \mathbf{V}^T \quad . \tag{5}$$

The pseudo-inverse is an $M \times N$ matrix, and is called an inverse because it produces an $M \times M$ identity matrix when it premultiplies **V**. Following Gelb[4], the pseudo-inverse is the Least Squares solution to Eq. 4. Note that the quantity $\mathbf{V}^T \mathbf{V}$ may be singular and not have an inverse in unusual cases where all of the outputs are not exercised by the test inputs. Premultiplying both sides of Eq. 4 by Eq 5 allows one to solve for the matrix, **A**:

$$\mathbf{A} = \left(\mathbf{V}^T \mathbf{V}\right)^{-1} \mathbf{V}^T \mathbf{F} \quad . \tag{6}$$

This solution applies to the case of data in row format as described by Eq. 4. For the case of data in column format where $\tilde{\mathbf{F}}$ is an $M \times N$ matrix of input forces and moments, and $\tilde{\mathbf{V}}$ is an $M \times N$ matrix of resulting voltages, Eq. 4 is rewritten as:

$$\tilde{\mathbf{F}} = \tilde{\mathbf{A}} \tilde{\mathbf{V}} \quad . \tag{7}$$

The solution for $\tilde{\mathbf{A}}$ is readily found to be:

$$\tilde{\mathbf{A}} = \tilde{\mathbf{F}} \tilde{\mathbf{V}}^T \left(\tilde{\mathbf{V}} \tilde{\mathbf{V}}^T\right)^{-1} \quad . \tag{8}$$

as can be seen by post multiplying both sides of Eq. 7 by the pseudo inverse $\tilde{\mathbf{V}}^T \left(\tilde{\mathbf{V}} \tilde{\mathbf{V}}^T\right)^{-1}$. Eq. 8 gives the interaction matrix in the form in which it is normally used. The relationship between Eqs. 6 and 8 is easily shown. Noting that $\tilde{\mathbf{F}} = \mathbf{F}^T$ and $\tilde{\mathbf{V}} = \mathbf{V}^T$, these expressions can be substituted into Eq. 7 to obtain

$$\mathbf{F}^T = \tilde{\mathbf{A}} \mathbf{V}^T \quad . \tag{9}$$

Taking the transpose of both sides of Eq. 9 gives:

$$\mathbf{F} = \left(\tilde{\mathbf{A}}\mathbf{V}^T\right)^T = \mathbf{V}\tilde{\mathbf{A}}^T \quad . \tag{10}$$

Comparison of Eq. 10 with Eq. 4 shows that $\tilde{\mathbf{A}} = \mathbf{A}^T$. This result can also be found by plugging $\tilde{\mathbf{F}} = \mathbf{F}^T$ and $\tilde{\mathbf{V}} = \mathbf{V}^T$ into Eq. 8 and simplifying.

Summarizing, the interaction matrix is found from Eq. 8; however, if the data is more conveniently arranged in row format, then the solution may also be obtained by using Eq. 6 and taking the transpose. This Least Squares solution can be modified to incorporate a weight function in order to favor data considered to be more reliable. The development of an interaction matrix with a weighted least squares technique may be found in the next section.

Weighted Least Squares Derivation (Pseudo Inverse)

Given a set of N instances of dynamometer measurements with M measurements for each instance, the data must satisfy:

$$\mathbf{F} = \mathbf{V}\mathbf{A} \quad , \tag{11}$$

where $\mathbf{F}$ is the set of forces and moments that produces the voltage outputs $\mathbf{V}$. $\mathbf{F}$ is an N x M matrix of input forces and moments, $\mathbf{V}$ is an N x M matrix of resulting voltages, and $\mathbf{A}$ is an M x M matrix. Note that in this formulation the data is in row format with each row representing a separate instance of the calibration process; therefore, $\mathbf{A}$ is the transpose of the interaction matrix.

The matrix formulation of the problem, while convenient, tends to obscure the fact that there are M independent Least Squares problems contained in Eq. 11, one for each axis of the dynamometer. The Least Squares solution for $\mathbf{A}$ will be illustrated by a derivation for one axis (column of $\mathbf{F}$); the derivations for the other axes are identical. Then, the final results for the M problems can be conveniently expressed by a single solution by again using matrix notation.

Denoting the first columns of $\mathbf{F}$ and $\mathbf{A}$ by $\mathbf{F}_1$ and $\mathbf{A}_1$, we can write the weighted error as:

$$\mathbf{E}_1 = \mathbf{W}(\mathbf{F}_1 - \mathbf{V}\mathbf{A}_1) \quad , \tag{12}$$

where $\mathbf{E}$ is an N x 1 vector of errors, $\mathbf{W}$ is an N x N diagonal matrix that weights the contributions of each data spot in the fit, $\mathbf{F}_1$ and $\mathbf{A}_1$ are N x 1 and M x 1 vectors, and $\mathbf{V}$ is the N x M voltage matrix. To compute the sum of the squared errors which is the minimization criterion for a Least Squares fit, one forms the quantity:

$$\mathbf{E}_1^2 = \mathbf{E}_1^T\mathbf{E}_1 \quad . \tag{13}$$

This cost function represents the weighted sum of squares of the errors between the $\mathbf{F}_x$ load input and the resulting $\mathbf{F}_x$ voltage measurement from the dynamometer. Now, the least squares solution $\mathbf{A}_1$ is found by minimizing $\mathbf{E}_1^2$ with respect to $\mathbf{A}_1$; that is, taking the derivative and setting it equal to zero:

$$
\begin{aligned}
\frac{\partial \mathbf{E}_1^2}{\partial \mathbf{A}_1} &= \frac{\partial}{\partial \mathbf{A}_1}\left[(\mathbf{F}_1 - \mathbf{V}\mathbf{A}_1)^T \mathbf{W}^T \mathbf{W}(\mathbf{F}_1 - \mathbf{V}\mathbf{A}_1)\right] = 0 \\
&= \frac{\partial}{\partial \mathbf{A}_1}\left[\mathbf{F}_1^T\mathbf{W}^T\mathbf{W}\mathbf{F}_1 - \mathbf{F}_1^T\mathbf{W}^T\mathbf{W}\mathbf{V}\mathbf{A}_1 - \mathbf{A}_1^T\mathbf{V}^T\mathbf{W}^T\mathbf{W}\mathbf{F}_1 + \mathbf{A}_1^T\mathbf{V}^T\mathbf{W}^T\mathbf{W}\mathbf{V}\mathbf{A}_1\right] = 0 \\
&= 0 - \left(\mathbf{F}_1^T\mathbf{W}^T\mathbf{W}\mathbf{V}\right)^T - \mathbf{V}^T\mathbf{W}^T\mathbf{W}\mathbf{F}_1 + \mathbf{V}^T\mathbf{W}^T\mathbf{W}\mathbf{V}\mathbf{A}_1 + \left(\mathbf{A}_1^T\mathbf{V}^T\mathbf{W}^T\mathbf{W}\mathbf{V}\right)^T = 0 \quad . \\
&= -2\mathbf{V}^T\mathbf{W}^T\mathbf{W}\mathbf{F}_1 + 2\mathbf{V}^T\mathbf{W}^T\mathbf{W}\mathbf{V}\mathbf{A}_1 = 0 \\
&\mathbf{V}^T\mathbf{W}^T\mathbf{W}\mathbf{V}\mathbf{A}_1 = \mathbf{V}^T\mathbf{W}^T\mathbf{W}\mathbf{F}_1
\end{aligned} \tag{14}
$$

Premultiplying both sides of the final result of Eq. 14 by the inverse of the left hand side yields:

$$
\mathbf{A}_1 = \left(\mathbf{V}^T\mathbf{W}^T\mathbf{W}\mathbf{V}\right)^{-1}\mathbf{V}^T\mathbf{W}^T\mathbf{W}\mathbf{F}_1 \quad . \tag{15}
$$

This is the solution for the first column of the **A** matrix. The derivation for the remaining axes of the dynamometer proceeds similarly. Therefore, for the i^{th} axis the solution becomes:

$$
\mathbf{A}_i = \left(\mathbf{V}^T\mathbf{W}^T\mathbf{W}\mathbf{V}\right)^{-1}\mathbf{V}^T\mathbf{W}^T\mathbf{W}\mathbf{F}_i \quad , \tag{16}
$$

where the subscript i refers to the respective column of the **F** and **A** matrices. Finally, matrix notation allows us to write the solution to all M least squares problems in a compact form as:

$$
\mathbf{A} = \left(\mathbf{V}^T\mathbf{W}^T\mathbf{W}\mathbf{V}\right)^{-1}\mathbf{V}^T\mathbf{W}^T\mathbf{W}\mathbf{F} \quad . \tag{17}
$$

This solution methodology may, of course, be used to derive the results of the unweighted method described in the previous section. Accordingly, if **W** is the identity matrix, Eq. 17 reduces to the result given in Eq. 6. The reader is reminded that this solution applies to the case of data specified in row format, which is not the usual manner in which the interaction matrix is used. As discovered earlier, the interaction matrix may be found as the transpose of the result given in Eq. 17. Thus, the final solution for the interaction matrix may be found to be:

$$
\widetilde{\mathbf{A}} = \mathbf{A}^T = \left[\left(\mathbf{V}^T\mathbf{W}^T\mathbf{W}\mathbf{V}\right)^{-1}\mathbf{V}^T\mathbf{W}^T\mathbf{W}\mathbf{F}\right]^T = \mathbf{F}^T\mathbf{W}^T\mathbf{W}\mathbf{V}\left(\mathbf{V}^T\mathbf{W}^T\mathbf{W}\mathbf{V}\right)^{-1} \quad , \tag{18}
$$

where the notation $\mathbf{F}^T$,for example, implies the transpose of the $N \times M$ matrix of data in row format. The result may be further simplified by dealing directly with the matrix $\mathbf{W}^2 = \mathbf{W}^T\mathbf{W}$ giving:

$$
\widetilde{\mathbf{A}} = \mathbf{F}^T\mathbf{W}^2\mathbf{V}\left(\mathbf{V}^T\mathbf{W}^2\mathbf{V}\right)^{-1} = \widetilde{\mathbf{F}}\,\mathbf{W}^2\,\widetilde{\mathbf{V}}^T\left(\widetilde{\mathbf{V}}\,\mathbf{W}^2\,\widetilde{\mathbf{V}}^T\right)^{-1} \quad , \tag{19}
$$

where $\mathbf{W}^2$ is the $N \times N$ diagonal matrix given by

$$
\mathbf{W}^2 = \begin{bmatrix} w_1^2 & 0 & 0 & 0 & \cdots & 0 \\ 0 & w_2^2 & 0 & 0 & \cdots & 0 \\ 0 & 0 & w_3^2 & 0 & \cdots & 0 \\ 0 & 0 & 0 & w_3^2 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \cdots & w_N^2 \end{bmatrix} . \tag{20}
$$

Typically, an interaction matrix that is computed by this method or the unweighted method given in the previous section is determined relative to the coordinate system used during calibration. Since calibration is normally carried out by the dynamometer manufacturer, the reference coordinate system is one normally used by the manufacturer and is not usually the required 2510 coordinate system used at NSWC. The following section describes a method to transform the interaction matrix relative to one coordinate system into that relative to another desired coordinate system.

Coordinate System Transformation

This section describes a method for the transformation of an interaction matrix determined relative to one coordinate system into another interaction matrix relative to a second coordinate system. Consider Fig. 4, which shows the dynamometer coordinate system (DCS) on the left and the standard coordinate system (NSWC) on the right.

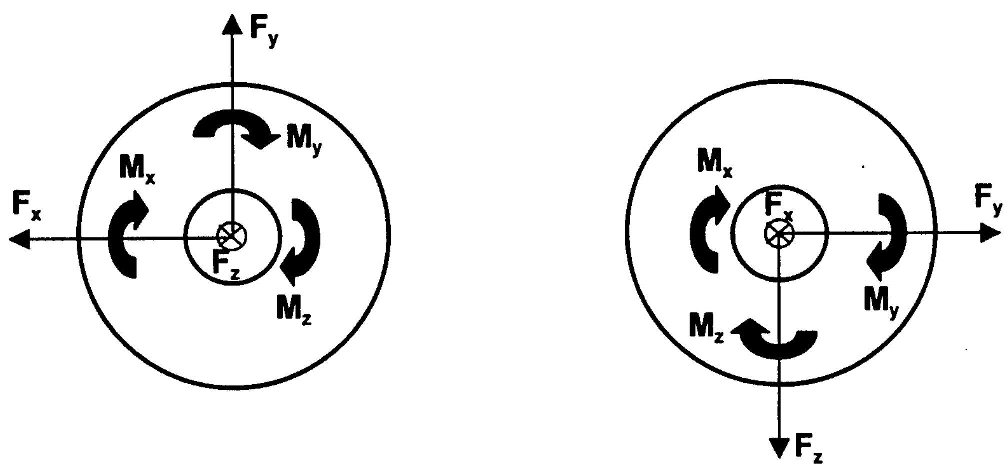

Fig. 4 Dynamometer coordinate system (DCS) on left and standard coordinate system on right.

The DCS force vector $\mathbf{F}$ written in NSWC (2510)[1,2] notation is on the left of Eq. 21, and the new force vector that we wish to obtain after transformation is $\widetilde{\mathbf{F}}$ which is shown on the right of Eq. 21:

$$\mathbf{F} = \begin{bmatrix} -F_y \\ -F_z \\ F_x \\ -M_y \\ -M_z \\ M_x \end{bmatrix} , \quad \tilde{\mathbf{F}} = \begin{bmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{bmatrix} . \tag{21}$$

The new force vector, $\tilde{\mathbf{F}}$, may be obtained by multiplying by a transformation matrix as shown in Eqs. 22 and 23:

$$\tilde{\mathbf{F}} = \mathbf{TF} \ . \tag{22}$$

$$\underset{\mathbf{T}}{\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}} \underset{\mathbf{F}}{\begin{bmatrix} -F_y \\ -F_z \\ F_x \\ -M_y \\ -M_z \\ M_x \end{bmatrix}} \underset{=}{=} \underset{\tilde{\mathbf{F}}}{\begin{bmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{bmatrix}} . \tag{23}$$

Similarly, the DCS voltage vector **V** (in $\mu V/V_{ex}$) written in NSWC notation is on the left of Eq. 24, and the new voltage vector that we wish to obtain after transformation is $\tilde{\mathbf{V}}$ which is shown on the right of Eq. 24. Note that we do not wish to change the polarity of the voltages because when the unit is wired for operation we will only want to swap the appropriate wires.

$$\mathbf{V} = \begin{bmatrix} V_{fy} \\ V_{fz} \\ V_{fx} \\ V_{my} \\ V_{mz} \\ V_{mx} \end{bmatrix} , \quad \tilde{\mathbf{V}} = \begin{bmatrix} V_{fx} \\ V_{fy} \\ V_{fz} \\ V_{mx} \\ V_{my} \\ V_{mz} \end{bmatrix} . \tag{24}$$

The new voltage vector, $\tilde{\mathbf{V}}$, may be obtained by multiplying by a transformation matrix as shown in Eqs. 25 and 26:

$$\tilde{\mathbf{V}} = \mathbf{SV} \ . \tag{25}$$

$$
\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}
\begin{bmatrix} V_{fy} \\ V_{fz} \\ V_{fx} \\ V_{my} \\ V_{mz} \\ V_{mx} \end{bmatrix}
=
\begin{bmatrix} V_{fx} \\ V_{fy} \\ V_{fz} \\ V_{mx} \\ V_{my} \\ V_{mz} \end{bmatrix} . \tag{26}
$$

$$
\mathbf{S} \qquad \mathbf{V} = \widetilde{\mathbf{V}}
$$

Notice that $\mathbf{S} = |\mathbf{T}|$. Now, if the original interaction matrix is denoted by **A**, then the desired transformed matrix will be called $\widetilde{\mathbf{A}}$. The interaction matrix **A** satisfies:

$$
\mathbf{F} = \mathbf{A}\mathbf{V} \ . \tag{27}
$$

Premultiply both sides of Eq. 27 by the transformation matrix **T** to get:

$$
\mathbf{T}\mathbf{F} = \mathbf{T}\mathbf{A}\mathbf{V} \qquad \text{or} \qquad \widetilde{\mathbf{F}} = \mathbf{T}\mathbf{A}\mathbf{V} \qquad \text{from Eq. 22} \ . \tag{28}
$$

Next notice that both sides of Eq. 25 may be premultiplied by $\mathbf{S}^{-1}$ to get:

$$
\mathbf{S}^{-1}\widetilde{\mathbf{V}} = \mathbf{S}^{-1}\mathbf{S}\mathbf{V} = \mathbf{V} \ . \tag{29}
$$

Then, Eq. 29 may be substituted into Eq. 28 to obtain:

$$
\widetilde{\mathbf{F}} = \mathbf{T}\mathbf{A}\mathbf{S}^{-1}\widetilde{\mathbf{V}} \ . \tag{30}
$$

Therefore, the transformed interaction matrix $\widetilde{\mathbf{A}}$ must be given by:

$$
\widetilde{\mathbf{A}} = \mathbf{T}\mathbf{A}\mathbf{S}^{-1} \qquad \text{to satisfy} \qquad \widetilde{\mathbf{F}} = \widetilde{\mathbf{A}}\widetilde{\mathbf{V}} \ . \tag{31}
$$

Finally, recalling that $\mathbf{S} = |\mathbf{T}|$, Eq. 31 may be written as

$$
\widetilde{\mathbf{A}} = \mathbf{T}\mathbf{A}|\mathbf{T}|^{-1} \ , \tag{32}
$$

where **T** is given by:

$$
\mathbf{T} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix} . \tag{33}
$$

Of course, **T** given above in Eq. 33 will depend upon the particular coordinate system in use by the dynamometer manufacturer; a coordinate system other than that shown on the left side of Fig. 4 will result in a different matrix for **T**. Once an interaction matrix relative to a desired coordinate system is obtained, one will find normalization of this matrix to be a useful procedure. The next section describes such a technique.

Normalization Technique

Normalization of the interaction matrix permits easy comparison of an interaction matrix from one dynamometer with that from another. A normalized matrix also allows one to easily estimate the size of the interaction (off-diagonal) terms relative to the diagonal terms, and to assess the penalty for replacing the full interaction matrix with M transformation coefficients (gains) obtained from the diagonal terms.

Consider a set of N instances of dynamometer measurements with M measurements for each instance. For the case of data in column format, **F** is an $M \times N$ matrix of input forces and moments, **V** is an $M \times N$ matrix of resulting voltages, and **A** is the $M \times M$ interaction matrix. The data must then satisfy:

$$\mathbf{F} = \mathbf{A}\mathbf{V} \ . \tag{34}$$

In this columnar format each row of the interaction matrix corresponds to the coefficients of the output data reduction equation. For example:

$$F_x = A_{11} V_{fx} + A_{12} V_{fy} + \ldots + A_{16} V_{mz} \ . \tag{35}$$

Therefore, the columns of the interaction matrix represent the coefficients required for a given axis in the reduction equation. The first column, for example, contains the coefficients needed to describe the F_x contribution. The normalization procedure proceeds by dividing each column of the interaction matrix by the diagonal element associated with that column. This will produce ones on the diagonal, and the off-diagonal terms may be interpreted as fractions (percentages) of the diagonal element in each column. The interaction matrix can then be written as:

$$\mathbf{A} = \tilde{\mathbf{A}}\mathbf{G} \quad \text{and} \quad \tilde{\mathbf{A}} = \mathbf{A}\mathbf{G}^{-1} \ . \tag{36}$$

where **A** is the original interaction matrix, $\tilde{\mathbf{A}}$ is the normalized interaction matrix and **G** is a diagonal matrix of transformation factors that convert the raw voltages to engineering units. Substituting Eq. 36 into Eq. 34 yields a two-step conversion process:

$$\mathbf{F} = \tilde{\mathbf{A}}\mathbf{G}\mathbf{V} \ , \tag{37}$$

where the voltages are first multiplied by **G** to transform them into engineering units, then the engineering units are in turn multiplied by the normalized interaction matrix to account for the interactions.

An example of a normalized interaction matrix, $\tilde{\mathbf{A}}$, is shown in Table 1:

1.00	-0.02	0.00	0.00	0.00	0.00
0.00	1.00	0.02	0.00	0.01	-0.02
0.00	-0.03	1.00	0.00	0.01	0.00
0.00	-0.01	-0.01	1.00	-0.01	0.01
0.01	0.00	0.01	0.00	1.00	0.02
0.00	0.01	0.00	0.00	-0.01	1.00

Table 1 Normalized interaction matrix.

The advantage of this approach is that $\tilde{\mathbf{A}}$ is now composed of coefficients that make the interactions readily apparent. For the above interaction matrix, the F_x output can be computed by:

$$F_{x\,(\text{with interactions})} = F_x - 0.02F_y \quad . \tag{38}$$

The inverse of the gain matrix, $\mathbf{G}^{-1}$, for this example is shown in Table 2:

1.53	0.00	0.00	0.00	0.00	0.00
0.00	-5.95	0.00	0.00	0.00	0.00
0.00	0.00	-5.95	0.00	0.00	0.00
0.00	0.00	0.00	5.71	0.00	0.00
0.00	0.00	0.00	0.00	-8.40	0.00
0.00	0.00	0.00	0.00	0.00	-8.40

Table 2 Inverse of the gain matrix.

The original interaction matrix, **A**, prior to normalization is provided in Table 3:

0.652	0.0039	0.0008	-0.0004	0.0005	0.0003
-0.0007	-0.168	-0.0028	-0.0001	-0.0006	0.0019
-0.0017	0.0045	-0.167	-0.0002	-0.0017	-0.0005
0	0.0018	0.002	0.175	0.0007	-0.001
0.004	0.0002	-0.001	0.0008	-0.119	-0.0019
-0.0027	-0.0011	0.0003	-0.0007	0.0007	-0.119

Table 3 Original interaction matrix.

The subject of rotating dynamometer's is discussed next. The following two sections will cover issues such as offset angle and weight and zero compensation for dynamometers that rotate.

Rotating Dynamometer Offset Angle

When instantaneous measurements of forces and moments which are acting on a rotating object, such as a propeller, are desired, a dynamometer is mounted so that it rotates along with the object. Careful consideration must be given to the frequency response of the coupled system. Miller[3] suggests that the

dynamometer must have a relatively flat frequency response extending from the lowest shaft frequency to several times the highest propeller blade frequency, and that it must also be able to measure the steady components of the forces and moments. Resonances that may exist in the coupled system that fall within this working range must be identified by a dynamic calibration.

Forces and moments measured by a rotating dynamometer are determined relative to a coordinate system fixed within the dynamometer and defined by the placement of the various strain gages within the unit. Therefore, as measurements are acquired at successive time intervals, the reference coordinate system is rotating. This is inconvenient. One would prefer that measurements at successive time intervals be referred to a fixed set of axes. Typically, when mounting the unit, the x-axis (thrust axis) of the dynamometer is oriented along the axis of rotation. If the x-axis of the fixed set of reference axes for the vehicle is also oriented along the axis of rotation such that the origins of the two systems are coincident, then the transformation from the rotating system to the fixed system is particularly simple and requires knowledge only of the instantaneous angle, θ, that exists between corresponding axes of the two systems.

Therefore, an additional task unique to rotating dynamometers is the determination of the instantaneous angular position of the device. Ordinarily, this is a trivial matter as the propeller angular position is measured; however, after installation of the rotor dynamometer, its initial orientation relative to this measured angular position is unknown. This section describes a method for the determination of this offset angle, θ_{off}.

Typically, weights are hung on the prop and the prop is rotated with the weights in place. The rotor dynamometer responds with measured force components along the F_y and F_z axes which vary in magnitude as the prop is rotated. When the rotating coordinate system of the dynamometer is precisely aligned with the F_z axis vertical, then the entire weight will be sensed along the F_z axis and none will be measured along the F_y axis. However, if a small offset angle exists, then the F_y axis will respond with a nonzero quantity. This measurement can be used to determine the offset angle. This idealized description is complicated by the fact that the dynamometer also measures its own weight and that of the prop even with no additional weight applied. Therefore, several measurements are required.

A given prop position is acquired using the current prop position measurement and the corresponding prop position readout. Adjust the prop position until it is at a nominal zero degrees. Measure the prop F_y and F_z loads, the transformed F_y and F_z loads and the prop position with and without the added weight applied. The offset angle is then determined from:

$$\theta_{off} = \sin^{-1}\left[\frac{F_{y2} - F_{y1}}{W}\right] , \tag{39}$$

where W is the applied weight, F_{y2} is the computed F_y side force measurement (interaction matrix and rotation matrix applied) with the weight applied at zero degrees of prop position readout, and F_{y1} is the

computed F_y side force measurement (interaction matrix and rotation matrix applied) without the weight applied at zero degrees of prop position readout.

The procedure should then be repeated with the prop adjusted to some desired nonzero angle. Once again, calculate the offset for the prop position readout by taking the difference between the computed F_y outputs with and without the weight for the given prop position. A second computation of the offset angle will then serve as a check on the previously attained value.

Rotating Dynamometer Weight and Zero Compensation

Both rotor and stator dynamometers will measure the weight of the respective propeller parts in water. For the case of the rotor dynamometer, the propeller weight will result in a sinusoidal *noise* that will have an amplitude equal to the propeller weight. This is a result of the fact that forces and moments are measured with respect to the coordinate system rotating with the dynamometer. This weight contribution must be removed in order to measure only those forces and moments exerted by the fluid on the rotor or stator, respectively.

The method for removal of component weights requires that measurements be referred to a coordinate system fixed within the vehicle and denoted the body coordinate system. For a fixed stator dynamometer, no transformation is required, whereas for the rotor dynamometer, a transformation from the rotating dynamometer coordinate system to the fixed body coordinate system is required. This transformation from the rotating system to the fixed system is particularly simple and requires knowledge only of the instantaneous angle, θ, that exists between corresponding axes of the two systems. The transformation is given by:

$$\mathbf{T}_{rot} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 & 0 & 0 \\ 0 & \sin\theta & \cos\theta & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cos\theta & -\sin\theta \\ 0 & 0 & 0 & 0 & \sin\theta & \cos\theta \end{bmatrix} . \tag{40}$$

Recall that for a set of N instances of dynamometer measurements with M measurements for each instance with the data grouped in column format, $\mathbf{F}$ is an M x N matrix of input forces and moments, $\mathbf{V}$ is an M x N matrix of resulting voltages, and $\mathbf{A}$ is the M x M interaction matrix. The data must then satisfy:

$$\mathbf{F} = \mathbf{A}\mathbf{V} \ . \tag{41}$$

Application of the transformation in Eq. 40 to rotor dynamometer data yields forces and moments referred to the body coordinate system of the vehicle:

$$\mathbf{F}_{bc} = \mathbf{T}_{rot}\,\mathbf{A}\,\mathbf{V} \quad . \tag{42}$$

Note that a similar approach must be applied to any rotating dynamometer such as those used for appendages on the vehicle.

Now, the weight vector for the propeller (or appendage) will always point vertically downward when referred to an inertial coordinate system fixed at the surface of the fluid in which the vehicle is operating. However, the vehicle itself may assume any arbitrary attitude while maneuvering. Therefore, the weight vector must be transformed from the fixed inertial system into the body coordinate system moving with the vehicle. This transformation requires knowledge of the Euler angles: $\varphi, \theta,$ and ψ (roll, pitch and yaw) which describe the orientation of the vehicle relative to the fixed inertial coordinate system at a given instant in time.

The transformation matrix may be found in the SNAME paper[2] and is given by:

$$\mathbf{T}_{euler} = \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ \sin\varphi\sin\theta\cos\psi - \cos\varphi\sin\psi & \sin\varphi\sin\theta\sin\psi + \cos\varphi\cos\psi & \sin\varphi\cos\theta \\ \cos\varphi\sin\theta\cos\psi + \sin\varphi\sin\psi & \cos\varphi\sin\theta\sin\psi - \sin\varphi\cos\psi & \cos\varphi\cos\theta \end{bmatrix} \quad . \tag{43}$$

Applying this transformation to the weight vector yields:

$$\mathbf{W}_{bc} = \mathbf{T}_{euler} \begin{bmatrix} 0 \\ 0 \\ W \end{bmatrix} \quad . \tag{44}$$

At this point, dynamometer measurements and the weight vector have been transformed into the body coordinate system affixed to the vehicle. Force components measured by the dynamometer may be corrected by simply subtracting the weight components. Moment components are corrected by subtracting the components of the vector $\mathbf{R} \times \mathbf{W}_{bc}$, where $\mathbf{R}$ is the distance vector from the dynamometer origin to the point of action of the propeller (or appendage) weight. Summarizing, then, each instance of dynamometer measurements can be corrected to remove the influence of the weight of the propeller (or appendage) as follows:

$$\begin{bmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{bmatrix}_{No\,W} = \begin{bmatrix} F_{bc\,x} - W_{bc\,x} \\ F_{bc\,y} - W_{bc\,y} \\ F_{bc\,z} - W_{bc\,z} \\ M_{bc\,x} - (\mathbf{R} \times \mathbf{W}_{bc})_x \\ M_{bc\,y} - (\mathbf{R} \times \mathbf{W}_{bc})_y \\ M_{bc\,z} - (\mathbf{R} \times \mathbf{W}_{bc})_z \end{bmatrix} \quad . \tag{45}$$

Repeating this correction for each of the N instances of dynamometer measurements yields the M x N matrix of forces and moments referred to the body coordinate system and denoted $\mathbf{F}_{No\,W}$.

For rotating dynamometers the subject of zero compensation is somewhat more complicated than for those that do not rotate. In general, for a non-rotating instrument, the readings of the instrument at a zero condition must be subtracted from all other measurements made by that instrument during data collection. However, for a rotating dynamometer, the rotation matrix $\mathbf{T}_{rot}$ used to transform the measurements to body coordinates is a nonlinear time-varying transformation. The procedure for zero compensation in this case is as follows.

With the vehicle at rest, so that the fluid exerts no forces and moments upon the propeller (or appendage), acquire an instance of dynamometer measurements and correct the components to remove the influence of propeller (or appendage) weight as described above. Then, the data should satisfy:

$$\mathbf{F}_{No\,W} = \mathbf{T}_{rot}\,\mathbf{A}\,\mathbf{V}_0 \quad , \tag{46}$$

where $\mathbf{V}_0$ is a vector of dynamometer output voltages with zero forces and moments applied to the dynamometer. If nonzero, then these voltages represent adjustments that must be subtracted from the dynamometer outputs when acquiring data. The vector $\mathbf{V}_0$ can be found from:

$$\mathbf{V}_0 = \mathbf{A}^{-1}\,\mathbf{T}_{rot}^{\;-1}\,\mathbf{F}_{No\,W} = \mathbf{A}^{-1}\,\mathbf{T}_{rot}^{\;T}\,\mathbf{F}_{No\,W} \quad . \tag{47}$$

Note that both transformation matrices, $\mathbf{T}_{rot}$ and $\mathbf{T}_{euler}$, are orthogonal which implies that they satisfy:

$$\mathbf{T}^{-1} = \mathbf{T}^{T} \quad . \tag{48}$$

The next section describes the equations that are required to use two dynamometers about a common reference center. This might be required when the loads to be measured along one or more axes exceed the range of one dynamometer alone.

Equations for Utilizing Two Six Degree-of-Freedom Dynamometers

This section describes the equations needed to resolve the output of two six-degree-of-freedom dynamometers about a common reference center. Each of the six-degree-of-freedom dynamometers is a transducer capable of simultaneously measuring three force components, denoted by F_x, F_y, and F_z and three moments, denoted by M_x, M_y, and M_z.

The simplest and most frequently utilized case involves two dynamometers that are aligned along their F_x axis. The desired reference point for the resolved output is a point mid-way between the gages. This case is shown in Fig. 5 where the dynamometer on the left is denoted as the forward gage and the dynamometer on the right as the aft gage. The reference point is a distance d from each of the dynamometers.

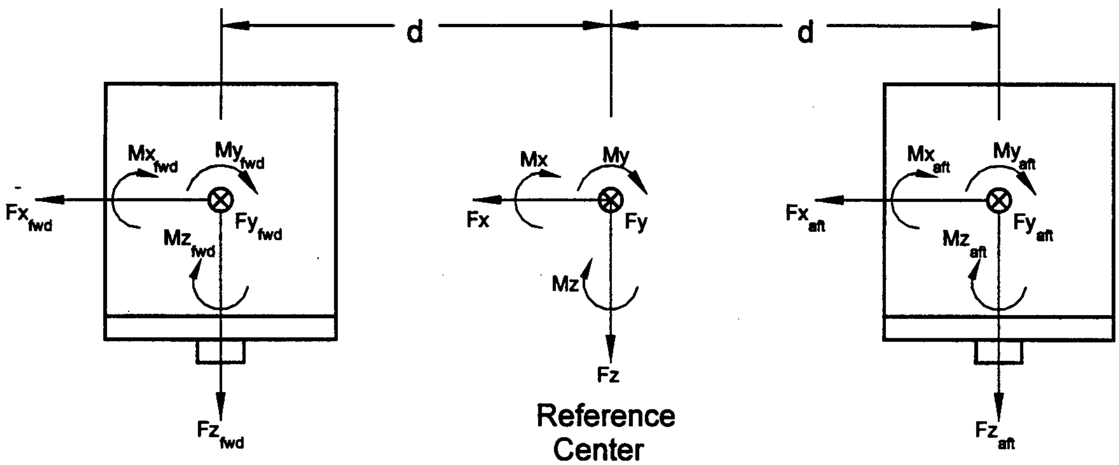

Fig. 5 Two dynamometers aligned along their F_x axis and equidistant from the reference center. The y-axis is into the paper.

The equations to obtain the forces and moments about the reference point are:

$$\begin{aligned}
F_x &= F_{x_{fwd}} + F_{x_{aft}} \\
F_y &= F_{y_{fwd}} + F_{y_{aft}} \\
F_z &= F_{z_{fwd}} + F_{z_{aft}} \\
M_x &= M_{x_{fwd}} + M_{x_{aft}} \\
M_y &= M_{y_{fwd}} + M_{y_{aft}} + d\left(F_{z_{aft}} - F_{z_{fwd}}\right) \\
M_z &= M_{z_{fwd}} + M_{z_{aft}} + d\left(F_{y_{fwd}} - F_{y_{aft}}\right)
\end{aligned} \tag{49}$$

The most general case involves two dynamometers that are offset from each other along all three primary axes. If x_1, y_1 and z_1 represent the offsets from the forward gage to the desired reference point and x_2, y_2 and z_2 represent the offsets from the aft gage, then the equations to obtain the forces and moments about the desired reference point are given by:

$$\begin{aligned}
F_x &= F_{x_{fwd}} + F_{x_{aft}} \\
F_y &= F_{y_{fwd}} + F_{y_{aft}} \\
F_z &= F_{z_{fwd}} + F_{z_{aft}} \\
M_x &= M_{x_{fwd}} + M_{x_{aft}} + z_1 F_{y_{fwd}} - y_1 F_{z_{fwd}} + z_2 F_{y_{aft}} - y_2 F_{z_{aft}} \\
M_y &= M_{y_{fwd}} + M_{y_{aft}} - z_1 F_{x_{fwd}} + x_1 F_{z_{fwd}} - z_2 F_{x_{aft}} + x_2 F_{z_{aft}} \\
M_z &= M_{z_{fwd}} + M_{z_{aft}} + y_1 F_{x_{fwd}} - x_1 F_{y_{fwd}} + y_2 F_{x_{aft}} - x_2 F_{y_{aft}}
\end{aligned} \tag{50}$$

If in Eq. 50 the following substitutions are made:

$$y_1 = y_2 = z_1 = z_2 = 0, \quad x_1 = -d \quad \text{and} \quad x_2 = d \tag{51}$$

then one can see that Eq. 50 reduces to Eq. 49.

The final section describes how one may determine the uncertainty in dynamometer measurements for the most common application. This example will then permit the reader to extend the uncertainty calculations to incorporate any of the other topics presented in this paper in a straightforward manner.

Data Uncertainty

The starting place for computation of uncertainty in dynamometer measurements is to determine the uncertainty present in the applied forces or moments to the dynamometer and in the output from the dynamometer during static calibration of the device. This fundamental raw data is obtained as described above in the calibration procedure. All of the remaining steps make use of this raw data; uncertainties in data at any subsequent step are calculated by propagating the uncertainties in the calibration data through the various governing equations at each step. Recall that the purpose of the calibration procedure was to produce a *linear* approximation of the behavior of the dynamometer by computing the interaction matrix, $\mathbf{A}$. Clearly then, the next goal of the uncertainty analysis must be to determine the uncertainty that has propagated into each element of the interaction matrix. Then, armed with this information, one can then easily determine the uncertainty in future measured forces and moments from the dynamometer by propagating the uncertainty in the interaction matrix and in measured output voltages into the reduction equation:

$$\mathbf{F} = \mathbf{VA} \ . \tag{52}$$

Following Coleman and Steele[5], uncertainty in a measurement consists of bias error and precision error. Bias error is a constant, systematic error, whereas, precision error is the random contribution often referred to as repeatability error. Considering the forces and moments applied to the dynamometer during calibration, analysis of the sources of bias error will depend upon the calibration method employed. For example, consider a calibration stand using weight pans connected to cables passing through pulleys to apply the loading to the dynamometer. If the calibration stand has not been accurately leveled, applied forces and moments will not be as expected. If various geometrical distances associated with the calibration stand are inaccurately measured, applied forces and moments will be affected by this inaccuracy. If the weights used to load the pans have drifted such that their true weight is not their stated weight, additional bias errors will creep into the calibration. If the pulleys do not have ideal, *frictionless* bearings, the true applied forces and moments will be altered by pulley resistance. Variation in physical constants can lead to bias errors; this is often the case when thermal variations are present. Incorrect experimental methods can lead to bias errors. For example, application of forces and moments during calibration in ascending or descending order as opposed to a random order can lead to a bias uncertainty. Identification of elemental bias errors for other types of dynamometer calibration equipment, such as load cells, would proceed in a similar fashion.

Sources of bias error are also present in the output from the dynamometer. Referring back to Eq. 2, repeated below for convenience, biases may be present in the A/D converter, the amplifier gain, and the applied excitation voltage. For example, a quantization bias error in the analog-to-digital converter is usually taken to be one-half of the least significant bit. Biases from the amplifier and power supply should be identified from manufacturer's specifications and should include factors such as gain, linearity and zero errors.

$$\frac{\mu V}{V_{ex}} = bits * \left(\frac{20}{4096}\frac{V}{bit}\right) * \left(1000000\frac{\mu V}{V}\right) * \left(\frac{1}{Gain}\right) * \left(\frac{1}{V_{ex}}\right) , \tag{2}$$

A detailed accounting of the sources of bias error is often difficult and requires a careful analysis of the calibration device, physical constants and geometrical data, associated experimental equipment and calibration procedures. Because of the expense that such a thorough examination may entail, one should perform a cursory investigation to estimate the order of magnitude of the biases. If all biases are negligible when compared to precision uncertainties, then clearly precision uncertainty will dominate, and setting the biases to zero will not significantly alter the calculation.

Precision error is determined by repetition. For the dynamometer calibration device, the ability to *repeatedly* apply a given force or moment must be quantified. If F_i is a force component applied by the calibration device during the i^{th} repetition, and N is the total number of measurements (spots) by an independent force gage (calibration standard), then compute a mean and a standard deviation using:

$$\overline{F} = \frac{1}{N}\sum_{i=1}^{N} F_i \quad \text{and} \quad S_F = \left[\frac{1}{N-1}\sum_{i=1}^{N}\left(F_i - \overline{F}\right)^2\right]^{1/2} . \tag{53}$$

A 95% confidence estimate of the precision uncertainty at a specific magnitude of applied force is estimated as:

$$P_{\bar{F}} = \frac{tS_F}{\sqrt{N}} , \tag{54}$$

where t represents a value drawn from the Student's t-distribution for a 95% confidence level and $\nu = N-1$ degrees of freedom. Beware, if N is small, then a 95% confidence level estimate will be large due to the paucity of the data; 5 to 10 trials should be sufficient to characterize the uncertainty level for a given force magnitude. One must perform this computation for a selection of force magnitudes throughout the dynamic range of the device. Similarly, the precision uncertainty of the output from the dynamometer, expressed in $\mu V/V_{ex}$, should be determined from repetitions with the same applied force or moment combination for a selection of force magnitudes throughout the dynamic range of the device.

Summarizing, a calibration of a six-degree-of-freedom dynamometer will yield an N x M array of applied force and moment components, **F**, and an N x M array of corresponding output voltages, **V**. (Note that these arrays have been written in row format, for convenience.) To each element of the **F** and **V** arrays there will be a corresponding bias and precision uncertainty that has been determined as described above. One may find, for example, that the bias and precision errors vary with force magnitude and are quite different for each axis. This is the most general case requiring distinct bias and precision uncertainties for each element of the **F** and **V** arrays. Conversely, one may find that the bias and precision errors are insensitive to force magnitude and are approximately the same for each axis; in this case, a single value of bias and precision uncertainty may be used for each element of the **F** and **V** arrays.

To arrive at an overall uncertainty, U_{ij}, for the i^{th} instance (of N instances) of the j^{th} applied force component (of M components) one combines the bias uncertainty with the precision uncertainty. The root-sum-square (RSS) method is most often used and is given by

$$U_{ij} = \sqrt{B_{ij}^2 + P_{ij}^2} , \tag{55}$$

with similar notation for the output voltages. This uncertainty is considered to be a 95% coverage estimate when B and P are 95% confidence values.

Next, consider how uncertainty propagates into a result. A general formula[5] for the uncertainty, U_R, which propagates into a result, R, from uncertainties in M different variables, X_i; $i = 1, \ldots, M$, where $R = R(X_1, X_2, \ldots, X_M)$ is given by:

$$U_R = \left[\left(\frac{\partial R}{\partial X_1} U_{X_1} \right)^2 + \left(\frac{\partial R}{\partial X_2} U_{X_2} \right)^2 + \ldots + \left(\frac{\partial R}{\partial X_n} U_{X_n} \right)^2 \right]^{1/2} . \tag{56}$$

When applying this formula to dynamometer measurements, we must determine how uncertainties in the calibration data propagate into each element of the interaction matrix and into future measured forces and moments. Recall that the solution for the interaction matrix is

$$\mathbf{A} = \left(\mathbf{V}^T \mathbf{V}\right)^{-1} \mathbf{V}^T \mathbf{F} \ . \tag{6}$$

If we consider only one dynamometer axis at a time (one column of **A** and **F**), then Eq. 6 can be rewritten as

$$\mathbf{V}^T \mathbf{V} \mathbf{A}_i = \mathbf{V}^T \mathbf{F}_i \ . \tag{57}$$

which is the classic form of the *normal equations* for a Least Squares fit problem. In this case, the basis functions of the fit are the six output voltages from the dynamometer *vis a vis* the more commonly employed monomials. Therefore, to determine the bias and precision uncertainty propagated into each column of the interaction matrix, one must determine how uncertainty in input data propagates into the coefficients of a Least Squares fit. Similarly, to determine the uncertainty present in future measured forces and moments from the fit (Eq. 52), one must understand how uncertainty propagates through a Least Squares fit into the output. This is done by simply applying Eq. 56 to the appropriate equations. The primary difficulty when using Eq. 56 is the calculation of the partial derivatives; a simple example will be used to make this clearer.

For example, consider the calculation of the uncertainty propagated into the slope, *m*, for a linear least squares fit of the form, $y = mx + b$. Recall that this slope depends on *N* pairs of input data (x_i, y_i) as described by

$$m = \frac{N\sum_{i=1}^{N} x_i y_i - \sum_{i=1}^{N} x_i \sum_{i=1}^{N} y_i}{N\sum_{i=1}^{N} x_i^2 - \left(\sum_{i=1}^{N} x_i\right)^2} \ . \tag{58}$$

The uncertainty in the slope depends upon the uncertainties in each of the abscissas and ordinates of the raw data used to construct the fit. In order to apply Eq. 56 to Eq. 58, one must determine the partial derivatives, $\frac{\partial m}{\partial x_i}$ and $\frac{\partial m}{\partial y_i}$, which are found to be

$$\frac{\partial m}{\partial x_i} = \frac{N y_i - \sum y_i - 2m\left(N x_i - \sum x_i\right)}{N\sum x_i^2 - \left(\sum x_i\right)^2} \quad \text{and} \quad \frac{\partial m}{\partial y_i} = \frac{N x_i - \sum x_i}{N\sum x_i^2 - \left(\sum x_i\right)^2} \ . \tag{59}$$

Then, the uncertainty in the slope may be found from

$$U_m = \left[\begin{array}{l} \left(\frac{\partial m}{\partial x_1} U_{x_1} \right)^2 + \left(\frac{\partial m}{\partial x_2} U_{x_2} \right)^2 + \ldots + \left(\frac{\partial m}{\partial x_N} U_{x_N} \right)^2 \\ + \left(\frac{\partial m}{\partial y_1} U_{y_1} \right)^2 + \left(\frac{\partial m}{\partial y_2} U_{y_2} \right)^2 + \ldots + \left(\frac{\partial m}{\partial y_N} U_{y_N} \right)^2 \end{array} \right]^{1/2} . \tag{60}$$

This example illustrates the fact that the analytical derivation of these partial derivatives for the simple case of a least squares fit of first order requires some tedious algebra and is clearly the most difficult step in applying the uncertainty calculation described by Eq. 56. One can imagine that these calculations could be performed for a second order (quadratic) fit with more difficulty, but that higher order fits would be too difficult. However, a method has been found by which these derivatives may be computed with ease for least squares curve and surface fits of any order, and implementing this method within a computer program enables the straightforward calculation of the propagation of uncertainty through such fits. The details of that method are too lengthy to reproduce here; however, the interested reader may find the information in the report by Hess and Smith.[6]

The example demonstrates how to propagate uncertainty into one of the coefficients of a least squares fit. For dynamometer calculations this example corresponds to the determination of the uncertainty propagated into one element of a given column of the interaction matrix. Given the uncertainties in the calibration data, a computer program can very rapidly perform these calculations. The report[6] also details how uncertainty may be propagated into future values determined from the fit. Thus, uncertainties in forces and moments measured by a calibrated dynamometer may be determined. Note that Eq. 60 deals with a general uncertainty, U. The propagation of precision uncertainty uses an identical equation with U replaced by P; however, the equation required to propagate bias uncertainties requires additional terms if biases are correlated. The details may be found in Hess and Smith[6] or in Coleman and Steele[5].

In a similar fashion, this uncertainty analysis may be extended to other quantities for which the governing equations appear in this paper. One proceeds by applying Eq. 56, and computing the required partial derivatives. In some cases, the analytic determination of the partial derivatives will be difficult or impossible. One may alternatively compute the derivatives numerically using forward, backward or central differences. The implementation of such a technique is usually referred to as a *jitter* program[5].

Conclusions

All of the techniques described in this paper have been tested and found to give good results. These methods have found use with both rotating and fixed dynamometers that have been employed on captive models and on free-running radio-controlled models during testing conducted within the Maneuvering and Control Department.

Appendix

Interaction Matrix Update from AMTI

As procedure dictates for a recent set of captive model experiments, the rotor and stator dynamometers were sent back to the dynamometer manufacturer (AMTI) for an extensive re-calibration involving 53 data points. The calibration results were delivered complete with interaction matrices. When the calibration results were checked by using the interaction matrices to recover the calibration data, large errors were discovered. AMTI was notified, and after investigation, they discovered an error in their computer program that generates the interaction matrices. We requested updated interaction matrices for all of the AMTI dynamometers in our possession. In accordance with ISO 9000 procedures, this appendix addresses the errors that are indicated by this factory update.

A spreadsheet was constructed that contains a comparison of uncorrected and corrected interaction matrices for all of the rotor and stator dynamometers that have been used for various experiments. A copy of that spreadsheet is provided below as Table 4. In each case the corrected matrix appears first followed by the original uncorrected matrix from AMTI. The matrices have been normalized for easy comparison using the procedure described previously. The last two comparisons contain three matrices with the third matrix developed using the pseudo-inverse method discussed in a previous section. The corrected matrices obtained from AMTI do not use the pseudo-inverse method; instead, they use a subset of the calibration data. Therefore, the final comparisons serve as a check on the corrected matrices from AMTI.

NSSN ROTOR GAINS		NORMALIZED 4/00 INTERACTION MATRIX FOR M3792					
1.54321		1.00	-0.02	-0.01	-0.02	-0.02	0.01
-5.95238		0.00	1.00	0.01	0.01	0.00	-0.01
-5.95238		0.00	0.01	1.00	-0.01	0.01	0.01
5.68182		0.00	0.01	-0.03	1.00	0.01	0.00
-8.40336		0.01	0.01	0.01	0.01	1.00	0.00
-8.40336		0.00	0.01	-0.01	0.00	0.01	1.00
GAINS	**% CHG.**	**OLD INTERACTION MATRIX FOR M3792**					
1.54799	-0.3096	1.00	-0.04	-0.02	-0.05	-0.10	0.04
-5.95238	0.0000	0.00	1.00	0.01	0.01	-0.01	-0.02
-5.98802	-0.5988	0.00	0.01	1.00	-0.01	0.02	0.02
5.68182	0.0000	0.00	0.01	-0.03	1.00	0.02	0.00
-8.40336	0.0000	0.00	0.00	0.00	0.01	1.00	0.00
-8.33333	0.8333	0.00	0.01	0.00	0.00	0.01	1.00

NSSN STATOR

GAINS	NORMALIZED 4/00 INTERACTION MATRIX FOR M3795					
1.24224	1.00	-0.02	-0.02	0.00	0.01	0.00
-4.60829	0.00	1.00	-0.01	0.00	0.00	0.06
-4.69484	0.00	0.01	1.00	0.00	-0.05	0.00
2.58398	0.00	0.01	-0.01	1.00	0.00	-0.01
-3.31126	0.00	-0.02	0.02	0.01	1.00	-0.01
-3.25733	0.01	0.02	-0.04	0.01	0.01	1.00

GAINS	% CHG.	OLD INTERACTION MATRIX FOR M3795					
1.24224	0.0000	1.00	-0.07	-0.09	-0.01	0.03	0.01
-4.60829	0.0000	0.00	1.00	-0.01	0.00	0.00	0.06
-4.69484	0.0000	0.00	0.01	1.00	0.00	-0.05	0.00
2.58398	0.0000	0.00	0.01	-0.01	1.00	0.00	-0.01
-3.31126	0.0000	0.00	-0.02	0.02	0.01	1.00	-0.01
-3.25733	0.0000	0.00	0.02	-0.04	0.00	0.01	1.00

ONR
BODY 1

GAINS	NORMALIZED 4/00 INTERACTION MATRIX FOR M3849					
1.53374	1.00	-0.02	0.00	0.00	0.00	0.00
-5.95238	0.00	1.00	0.02	0.00	0.01	-0.02
-5.98802	0.00	-0.03	1.00	0.00	0.01	0.00
5.71429	0.00	-0.01	-0.01	1.00	-0.01	0.01
-8.40336	0.01	0.00	0.01	0.00	1.00	0.02
-8.40336	0.00	0.01	0.00	0.00	-0.01	1.00

GAINS	% CHG.	OLD INTERACTION MATRIX FOR M3849					
1.53374	0.0000	1.00	-0.05	-0.01	0.00	-0.02	-0.01
-5.95238	0.0000	0.00	1.00	0.02	0.00	0.01	-0.03
-5.98802	0.0000	0.00	-0.01	1.00	0.00	0.03	0.01
5.71429	0.0000	0.00	-0.01	-0.01	1.00	-0.01	0.02
-8.40336	0.0000	0.00	0.00	0.00	0.00	1.00	0.02
-8.40336	0.0000	0.00	0.00	0.00	0.00	-0.01	1.00

CAPTIVE MODEL ROTOR
M3665

GAINS	NORMALIZED 4/00 INTERACTION MATRIX FOR M3665					
1.57233	1.00	-0.02	-0.01	-0.01	0.00	0.00
-5.58659	0.00	1.00	0.00	0.00	0.01	0.06
-5.58659	0.00	0.00	1.00	0.00	-0.06	0.00
5.37634	0.00	-0.01	0.02	1.00	0.00	0.00
-8.40336	0.02	0.00	-0.01	0.00	1.00	0.01
-8.40336	0.00	-0.01	-0.02	0.00	0.01	1.00

GAINS	% CHG.	OLD INTERACTION MATRIX FOR M3665 (1/00)					
1.57233	0.0000	1.00	-0.04	-0.02	-0.02	0.01	-0.01
-5.58659	0.0000	0.00	1.00	0.00	0.00	0.02	0.12
-5.58659	0.0000	0.00	0.00	1.00	0.00	-0.12	0.01
5.37634	0.0000	0.00	-0.01	0.02	1.00	-0.01	0.01
-8.40336	0.0000	0.01	0.00	0.00	0.00	1.00	0.01
-8.40336	0.0000	0.00	0.00	-0.01	0.00	0.01	1.00

PSEUDO-INVERSE
INTERACTION MATRIX M3665

GAINS	% CHG.	PSEUDO-INV INTERACTION MATRIX FOR M3665					
1.56954	0.1773	1.00	0.00	0.00	-0.01	0.00	0.00
-5.54444	0.7545	0.00	1.00	0.00	0.00	0.01	0.06
-5.51236	1.3288	-0.01	0.01	1.00	0.00	-0.06	0.00
5.42294	-0.8668	0.00	-0.02	0.01	1.00	-0.01	0.00
-8.37161	0.3778	0.02	0.00	0.01	0.00	1.00	0.01
-8.37189	0.3745	0.03	-0.01	-0.01	0.00	0.01	1.00

CAPTIVE MODEL STATOR
M3738

GAINS	NORMALIZED 4/00 INTERACTION MATRIX FOR M3738					
1.29199	1.00	-0.01	-0.02	0.00	0.01	0.00
-4.71698	0.00	1.00	0.00	0.00	0.00	0.02
-4.73934	0.00	0.01	1.00	0.00	-0.02	0.00
2.61097	-0.01	0.01	-0.02	1.00	0.00	-0.01
-2.56410	0.01	0.00	0.00	0.01	1.00	-0.01
-2.54453	0.01	0.00	-0.02	0.01	0.02	1.00

OLD INTERACTION MATRIX FOR M3738 (2/03/00)							
1.29199		1.00	-0.04	-0.09	0.00	0.02	0.00
-4.71698		0.00	1.00	0.00	0.00	0.00	0.02
-4.73934		0.00	0.01	1.00	0.00	-0.02	0.00
2.61097		0.00	0.01	-0.02	1.00	0.00	-0.01
-2.56410		0.00	0.00	0.00	0.01	1.00	-0.01
-2.54453		0.00	0.00	-0.02	0.01	0.02	1.00

GAINS	% CHG.	PSEUDO-INV INTERACTION MATRIX FOR M3738					
1.28593	0.4694	1.00	0.00	-0.01	0.00	0.01	0.00
-4.67530	0.8836	0.00	1.00	0.00	0.00	0.00	0.02
-4.68274	1.1941	0.00	0.01	1.00	0.00	-0.02	0.00
2.62522	-0.5460	0.00	-0.02	-0.02	1.00	0.00	-0.01
-2.56102	0.1204	0.00	-0.02	0.01	0.01	1.00	-0.01
-2.56003	-0.6093	0.04	0.00	0.00	0.00	0.01	1.00

Table 4 Uncorrected and corrected interaction matrices.

The normalization procedure reduces the interaction matrices to a form where the analysis can be performed by inspection. The analysis involved two steps. The first step was to compare the new interaction matrices with those computed using the pseudo-inverse method. If large errors remain, then the AMTI interaction matrices still cannot be used. The above shows that the difference between the matrices computed using the pseudo-inverse method and the AMTI matrices is only greater than 2% for the Fx interaction into Mz. Identical matrices were not expected, as AMTI does not use all of the calibration data to produce their interaction matrices; however, their corrected matrices appear to be reasonable.

The second step in the analysis procedure was to compare the old and new interaction matrices. There are a number of interaction changes that are greater than 2%, and these are shown in red. A threshold level of 2% was chosen because the gages are specified as 2% devices. For experimental results that relied upon the old interaction matrices, these differences will have to be examined. Since the raw voltage data is available, the analysis can be performed again using the updated matrices. An examination of the interaction matrices given above shows that the first row (Thrust or Fx) has had a change in the side force Fz correction. For some of the other updated matrices, the Fz correction due to My and the Fy correction due to Mz have also been changed.

The conclusion to be drawn from this analysis is that the new AMTI procedure for computing the interaction matrix has been validated using the 56 point calibration data. The errors in the recovery of the calibration data can be maintained within the 2% accuracy specified by the manufacturer. Based on these calibration results, the updated interaction matrices should be used for the analysis of propeller data.

References

1. Gertler, M. and G. Hagen, "Standard Equations of Motion for Submarine Simulation," NSRDC Report 2510 (June 1967).

2. "Nomenclature for Treating the Motion of a Submerged Body Through a Fluid," Report of the American Towing Tank Conference prepared by the Hydromechanics Subcommittee of the Technical and Research Committee of the Society of Naval Architects and Marine Engineers, Technical and Research Bulletin No. 1-5 (Apr 1950).

3. Miller, Marlin, L., "Experimental Determination of Unsteady Propeller Forces," Proceedings of the Seventh Symposium on Naval Hydrodynamics, Rome, Italy, (August 25-30, 1968), pp. 255-289.

4. Gelb, Arthur, ed., Applied Optimal Estimation, MIT Press, Cambridge, Massachusetts (1974).

5. Coleman, Hugh W. and W. Glenn Steele, Experimentation and Uncertainty Analysis for Engineers, Second edition, John Wiley and Sons, New York (1999).

6. Hess, David E. and William E. Smith, "Uncertainty Analysis Applied to Least Squares Curve and Surface Fits," Hydromechanics Directorate Research and Development Report NSWCCD-50-TR-1998 / 012 (Dec 1998).

THIS PAGE INTENTIONALLY LEFT BLANK

Initial Distribution

Copies		Code	Name
1	NAVSEA		
	1	05H2	King
	1	05H	Merrill
1	AMTI		
	1	Carignan	
2	DTIC		

Center Distribution

Copies	Code	Name
1	50	Morgan
1	5200	Stenson
1	5200	Fu
1	5300	Pickett
1	5400	Remmers
1	5500	Motter
1	5600	Koh
1	5600	Abramson, B.
1	5600	Ammeen
1	5600	Bochinski
1	5600	Cubbage
1	5600	Feldman
1	5600	Faller
1	5600	Junghans
1	5600	Mackintosh
1	5600	Moran
1	5600	Roddy

THIS PAGE INTENTIONALLY LEFT BLANK